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1. Introduction

In an urban society, public services and public facilities play an increasingly influential role in shaping the quality of urban life. For the planner, the location, size, and composition of public facilities present some of the most serious questions regarding service quality and service user welfare: What kinds of services should be provided at a facility? At what scale should the facility be constructed? What kinds of factors should be considered in determining facility location (e.g., cost, neighborhood opposition, time frame, economies of scale and agglomeration)? From a more pragmatic point of view, it is also crucial to deal with the effectiveness of existing facilities--i.e., to reevaluate the existing public facility locations relative to changing patterns of accessibility.

Where a public service is intended to be directly available to consumers or users, it has been argued that the problem of accessibility plays a dominant role in the decision of where to locate the facility and its subsequent utilization (Harris, 1966; Hansen, 1959). This concern with accessibility has, in turn, given rise to a class of models using distance minimization as a basis of a decision criterion for determining the optimal location of a set of facilities to serve an urban region. These models share a common mathematical programming formulation which serves both to locate facilities and to allocate consumers to them (thereby determining the boundaries of the service areas for each facility), given a concomitant minimization of the cost of assigning consumers to given facilities.

Though these models have shortcomings, they can prove useful for evaluating the accessibility of existing public services when information is available on user needs and characteristics, as well as on general population characteristics. The present paper attempts to show how the use of these models can help planners develop strategies for changing or improving public services when they are combined with localized, small-area, continuously provided information. Note that the point we wish to stress does not concern the development of new methodologies for analyzing public facility locations, but rather that the use of more detailed information within existing methodologies increases their effectiveness as inputs to decision-making procedures.

2. Public Facility Location Models

Scott (1971) has characterized the facility location-allocation problem as follows: given n points distributed in a plane and m centroids to be located in the plane, find locations for the centroids and an allocation of each point to a centroid in order to optimize an objective function (e.g., minimize cost or maximize net

benefits). Several authors have addressed themselves to this problem and have developed a number of algorithms, with variations accounting for facility capacity, maximum distance constraints, and pre-specification of the number of centroids to be located (e.g., Hakimi, 1964; Hakimi, 1965; Cooper, 1967; Scott, 1971; Toregas, Swain, ReVelle, and Bergman, 1971).

When we turn to realistic problems of evaluating the existing pattern of locations of public facilities, however, we note several drawbacks to the mathematical programming methods. First, consideration of census tracts as user nodes (or even a small proportion of those tracts as possible centroid locations) escalates the dimensions of the problem into a major and costly undertaking. Furthermore, the programming approach usually involves a methodological framework based upon classical, static optimization theory. For problems concerning public facility location, it is not at all clear that an "optimal location," in this sense, is the most or only important objective. The distribution of demand for such facilities will almost certainly vary over time (though not necessarily in a predictable manner). And the introduction of stochastic optimization functions and estimates of future distribution only serves to complicate an already complex problem.

The principal problem associated with the use of the usual mathematical programming methods for the location of public facilities may be traced to its generalized framework in which the location and service area of all facilities are solved simultaneously. While such methods might be warranted in a new urban area designed for a proscribed population, it is not suited to more heterogeneous urban contexts in which private and public planning coexist and many of the facilities are already fixed. In such cases, the consideration of the location of public facilities must account for a temporal sequence of decisions constrained by predetermined, bounded catchment areas which are relatively independent of each other.

We therefore conclude that (i) optimal facility location cannot be treated as an end in itself, (ii) the usual kinds of approaches may not be worth the effort expended, and (iii) individual facility locations may be treated independently in present urban contexts. The reality of public facility location, then, is a dynamic, incremental process.

Now, given all these disclaimers and provisos, what sort of procedure can be employed to determine public facility locations? After all, some method needs to be (and will be) employed, if only to provide a benchmark index against which to evaluate and compare the effects of existing patterns of facilities. In this context, optimization procedures have, in fact, proved to be quite useful--particularly where user in-

formation is available over closely spaced temporal intervals and a trend of "optimal locations" for a given facility may be obtained. It is in this context that optimization methods will be employed in this study: given the yearly data on community health facility utilization collected in Wichita, Kansas, (Sedgwick County Department of Mental Health, 1974), "optimal locations will be employed as a basis for calculating the locational deviation of the actual facility locations from an optimal facility location; the trend itself is then used to provide information on the rate of change in user accessibility. Our contention is that, used in this way, optimization methods can offer a basis for comparing alternative planning strategies vis à vis public facility location.

3. A Simple Facility Location Technique

Planners wishing to keep records of and derive trends from the optimal location of a given public facility relative to its actual location first need to identify the criteria by which "optimality" is judged. Since public services are ideally planned and located with the "public good" in mind, we propose here to employ a singular, consumer-oriented criterion for facility location; note that a more realistic treatment would also include considerations of professional and staff convenience (accessibility), scale and agglomeration economies, and other location factors (e.g., competing land uses).

The consumer or user orientation is based explicitly on the distance between the users of the service and the service facilities. Two criteria may be employed: the minimization of the aggregate distance traversed (i.e., the median point in a network of nodes and arcs) or the average distance between users and the facility (i.e., the mean point or center of gravity of a network).¹ For an urban region, areas (such as census tracts) are thus reduced to nodes and their position plotted in relation to an arbitrary orthogonal coordinate system. Distances of nodes from each other and from the coordinates' zero-value are then required for calculation of the median and mean points, respectively, which can be based on one of two metrics: (a) a euclidean distance metric (which assumes that users travel directly and in a straight line between two nodes), or (b) a metropolitan metric which incorporates the kind of resistance introduced by an orthogonal street network. Each node is then weighted by the number of users originating there and the median and mean distances are calculated as follows:²

$$\text{Median Distance} = \text{Min} \left(\frac{\sum_{j=1}^n w_j d_{ij}}{\sum_{j=1}^n w_j} \right) \text{ for each } i = 1, \dots, n \text{ where } w_j = \text{the weighting of the user node } j, \text{ and } d_{ij} = \text{the distance between two user nodes } i \text{ and } j$$

$$\text{Mean} = \left(\frac{\sum_{j=1}^n w_j d_{ij}}{\sum_{j=1}^n w_j} \right) \text{ for each } i = 1, \dots, n$$

where d_j = the distance of user node j from the zero-value of the reference coordinate system

Now, in order to depict the trend in optimal location and in the deviation of optimal from actual location, the mean or median point must be calculated at regular intervals over a period of time. As noted above, the information requirements include the availability of data on number of service users by origin node and destination point (public facility location) over several years. Furthermore, due to the relative ease with which the deviation of the median location from the actual location of a given public facility can be interpreted as the difference in aggregate user distance travelled and the immediate implications for user accessibility understood, the median point location method was selected as the more appropriate measure. An extension of this general method is also employed: weighting of user nodes. In effect, the number of users originating at a given node (census tract) can be considered as a form of weight and, by further differentiating the population by other properties, additional kinds of weights may also be obtained. Our argument for applying these alternative weighting functions is as follows: in public facility location, concern with consumer distance and accessibility inevitably raises the question of relative ability to overcome distance, i.e., the relative accessibility of the user population. For example, the poor and the elderly, who have fewer resources to spend on transportation, generally can be considered to have low relative accessibility to public services at any given location. Availability of conventional public transportation may ameliorate the problem but it cannot change the essential inequity. Planners who are faced with determining facility locations or evaluating the effects of relative accessibility to presently located facilities must therefore bear in mind such considerations and should, where possible, weight the location problem in favor of those people with poor accessibility.

4. An Example: Outpatient Mental Health Facilities

Wichita, Kansas has a population of about 350,000 people and thus is representative of medium-sized American cities. While the city and the surrounding county as a whole define a single catchment area for community mental health services, inpatient and outpatient services have been partitioned into a north district and a south district with one center for each. Their locations are marked on the accompanying map. (See Figure 1.) The derivation of optimal locations of outpatient centers and comparison with existing facilities was therefore carried out for the two districts separately in order to determine the trend in the accessibility of services relative to users over several years of operation of the centers.

Using a metropolitan metric in order to reflect actual distances over an urban street pattern and its influence on intra-urban travel, and an orientation of the reference coordinate system in conformity with the north-south, east-west directions of the grid pattern of streets in Wichita, the median location of outpatient centers corresponding to each district's user distribution for the years 1971, 1972, and 1973 was calculated. The first set of calculations was based upon the use of weights reflecting only the number of users of outpatient services originating at a given census tract. The second set of calculations involved a form of "welfare weighting" of the nodes of origin.³ While such weights were necessarily averages reflecting the properties of the entire population of the given census tract, they serve to index the differential accessibility by user resources as well as distance.

Employing a unique set of census records that are updated yearly in Wichita, the welfare weighting was devised to take account of income and vehicle ownership in a multiplicative relationship such that the disadvantage of the poor with no vehicles as opposed to the poor with vehicles and the discrepancy between the poor and the well-off with one or more vehicles would be emphasized. Where a_j = the service users at node j , Y_j = the percentage of the population at node j with income less than \$5000, and v_j = the average number of vehicles owned per household in node j , the welfare weighting of any given node is $W_j = a_j / (100.0 - Y_j) v_j$.

The weighted and unweighted sets of optimal facility locations over time are plotted in Figure 1. The unweighted optimal location is shaded with horizontal lines and the weighted location with vertical lines. (Since the weighted locations for the south district change for the two years, the location for 1973 is shaded diagonally.) The actual location is cross-hatched. The deviation calculation involves the determination of the difference between the aggregate distance travelled by all users to the actual outpatient location minus that of the optimal location. The trend in the distance deviation from optimal location (both weighted and unweighted), as well as the trend in the masses at those locations, is provided in Table 1.

As can be seen from Table 1, the unweighted optimal location in both the north and south districts remains constant over the three years. However, there is a significant difference in the deviations of the optimal location from the actual location: users in the south district face a distance deviation nearly $2\frac{1}{2}$ times that faced by users in the north district. However, the picture with respect to the weighted optimal locations is less clear. First, while the optimal location in the north district is the same for the two years, this is not true for the south district. The optimal location in the latter case shifts from census Tract 32 to Tract 39 (i.e., the unweighted optimal location). This, of course, leaves us with the question: Does this observation result from an actual shift in the user population (or characteristics of that population), or does it reflect other

possible biases in the data? The answer can only come with further analysis of public service use based on data which is not restricted to the areal aggregations provided by the decennial census. Also, it should be noted that in the north district, the distance deviation for the weighted location is less than that for the unweighted one. By taking the welfare of users into consideration, we decrease the distance between the optimal and actual locations, i.e., we move the optimal location closer to the actual one. With respect to the south district, the deviation increases for 1972 and remains the same for 1973. Finally, notice that for both districts and for both the unweighted and weighted locations, the masses (or "weights") increase from year to year. This implies that, in proportion to the total number of users, the number of service users in more distant areas is getting larger.⁴ This probably reflects population growth (and therefore health service user growth) on the metropolitan fringes.

5. Conclusions and Policy Implications

Using outpatient mental health services in Wichita, Kansas as an example, it has been shown that elaborate mathematical programming methods need not be resorted to in order to calculate optimal facility locations, especially when facilities can be treated independently. A relatively simple location technique was employed in order to derive optimal facility location with respect to user accessibility and to measure the trend in user accessibility over time via location deviation measures.

In effect, by using methods akin to that described above, public service planners are in a position to know whether or not a service facility at a given location is readily accessible to the people who need the service the most and those who have the most difficulty overcoming spatial separation. Moreover, with the availability of data over time, the trend in accessibility (indexed by the location deviation trend) can be determined. In cases where accessibility continues to decrease over time, the planner may be faced with a number of response options, including: relocating the facility, locating more facilities, changing the boundaries of user areas, or helping to increase the mobility of either the service (e.g., via mobile home trailers, van, or the like) or the users, perhaps by subsidizing their trips to the facility or by providing a shuttle bus system (Lankford, 1974). These options can only be evaluated in the light of information on the relative accessibility of the user population to public facilities over time. The methodology provided here is one means of obtaining that information quickly and cheaply.

Footnotes

1. Properties of the median point include the representation of the point of minimum aggregate travel, location with relatively little influence from extreme user node locations, and location that is not invariant with respect to the coordinate system to which nodes are referred.

The median point is calculated by finding that user node which has the smallest total sum of distances from all other user nodes. Properties of the mean point include an invariance with respect to the coordinate system used for positional reference, but greater influence from extreme user node locations and less intuitive meaning attached to the interpretation of the optimal location, which is calculated by taking the average distance of all user nodes to the zero-value of an arbitrary reference coordinate system (Neft, 1966; King, 1969; Scott, 1971).

2. Note that since the calculation of the mean results in a single number, a distance which is either the constant sum of the x- and y-distances in metropolitan metric or the euclidean distance between the mean point and the zero-value, two calculations must be made utilizing a different coordinate system (e.g., shift in measurement of x-coordinate distances) in order to yield a "complementary" distance value whose line plotted with the former will intersect at the mean point. Alternatively, for the metropolitan metric, the mean value of each coordinate can be determined separately and then paired to determine the coordinates of the mean location. The determination of the median point is made simply by keeping track of that user node which achieves the minimum aggregate distance.

3. Data were available for 1972 and 1973 only. In addition, the data on income for 1973 were incomparable with those of 1972, so that the percentage of the population having incomes from \$4000 to \$7000 was calculated, and one-third of this percentage was added to the percentage having income under \$4000. This might introduce a bias in the findings.

4. A simple increase in the total number of users does not increase the mass. Hence, since the distances remain constant over time, the

changes in masses are due to proportionate changes in the user population.

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TABLE 1

South District					
		optimal location (CT)	actual location (CT)	deviation distance (ft)	mass
unweighted	1971	39	65	14362	14767.0
	1972	39	65	14362	15448.76
	1973	39	65	14362	15623.82
weighted	1972	32	65	22361	12229.17
	1973	39	65	14362	14739.94
North District					
		optimal location (CT)	actual location (CT)	deviation distance (ft)	mass
unweighted	1971	4	6	5817	13124.98
	1972	4	6	5817	13416.66
	1973	4	6	5817	13779.83
weighted	1972	12	6	5454	9665.71
	1973	12	6	5454	11302.05

Figure 1

